Calculus in the visual aspects of teaching architecture students

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ABSTRACT: The curriculum taught to architecture students is interdisciplinary, which results in tying engineering with the visual arts and social sciences. The Dean of the Faculty of Architecture at Cracow University of Technology, Jacek Gyurkovich, described the essence of teaching in his article devoted to teaching design: *The knowledge and skills concerning building technology, construction law regulations and graphical design presentation techniques are essential, and are integrated with design classes conducted as a part of semester-long design modules at each level of education* [1]. As a result, the authors here have presented an implementation of the objectives of the Faculty which employs the authors. This has been presented as the example of a term project concerning calculus, as a part of which a visual approach to solving problems is used alongside rigorous calculation methods. The project focuses largely on individual work and manual execution.

INTRODUCTION

The mathematics curriculum at technical universities is adapted to each study course. However, the instructor individually defines the thematic scope and number of tasks, and their form devoted to each topic. Various forms of teaching are used in the teaching of mathematics at the Faculty of Architecture of Cracow University of Technology (CUT), in accordance with the following maxim by Confucius: *I hear and I forget. I see and I remember. I do and I understand* [2].

Every subject is preceded by a multi-media lecture, which is also made available to students through an e-learning platform. Afterwards, selected tasks or projects, appropriately tailored to the subject, familiarise students with new problems. Classes are conducted in a manner that has students solve individual problems themselves, both on the blackboard and on their own. Problems and quizzes available on the e-learning platform consolidate knowledge gained in class. They are solved as homework assignments and are mandatory as part of preparation for written and oral tests.

> *That which you had to discover on your own leaves a path in your mind. One that you can traverse once again, if needed* [3].

The poster is a separate project that is prepared at the end of the semester. It is in line with the concept of teaching mathematics while using various teaching techniques and methods [4]. The form of the poster was introduced due to the specificity of the architecture study course, in which graphical presentation plays a primary role. The project is associated with calculus, but is meant to be a conclusion of the skills gained during the first semester, not only in mathematics, but also as a part of other modules. This article is devoted to discussing this project.

VISUAL APPROACH IN ARCHITECTURE

Mentions of a multidirectional approach to architecture appeared already in a theory by Vitruvius, in the form of durability, utility and beauty. Gronostajska, in her article on Vitruvius' theory in contemporary architecture of the twenty-first century, defined beauty as: …*the logic of decisions concerning shape, proportions, materials and colours that comprise the overall effect* [5].

Misiagiewicz appreciates the role of drawing in the creative process by saying:

> *The role of drawing in studying architecture can be compared to that of a prompter, or rather a guide, because it indicates the essential values of the idea, directs attention to all these shapes and moments that cannot be noticed too clearly due to information overload* [6].

One example of an architectural scholar who points to the significance of the visual approach as a means of communication in contemporary culture is Kiroff who points out that:

> *The emergence of visual culture creates the premise for adopting a pictorial, rather than textual view of the world and even literature studies have been forced to conclude that the* world-as-a-text *has been replaced by the* world-as-a-picture [7].

Kiroff also found two approaches to using graphical language in architecture, which are, specifically, the concrete and the abstract, which she explained as follows:

> *The group of the concrete graphic languages, or the first step of the ladder, comprises orthographic, isometric, oblique, and perspective projection. Three-dimensional modelling is considered to be at the end of the abstract-to-concrete ladder of graphic languages. The group of the abstract languages, or the second step of the ladder, comprises charts, graphs, diagrams and schematics. The architectural profession as a whole seems to limit itself mainly to the first step of the ladder - the concrete graphic languages* [7].

Further, Kiroff performs surveys among architects, investigating their mode of work, which makes it possible to conclude that graphics - described as concrete - are used exclusively throughout design work and within finished materials prepared for clients: *Visual thinking is separated into three behaviours; seeing, imaging and drawing* [8].

She conducted studies during which designers used the traditional sketch and the PTC Pro/ENGINEER software program to prepare a conceptual proposal of a shelf. Won was of the opinion that traditional methods and not the computer stimulate these three behaviours during design and lead to a variety of results [8]. Undoubtedly, the digital display of a model results in a more detailed and attractive result, in addition to making it easier to imagine an object. However, it was through the use of traditional drawing that the greatest number of conceptual proposals was obtained.

CALCULUS POSTER

The project featuring the poster is introduced in the form of a lecture that presents the major concepts and assumptions of the project. It features formulae that make it possible to determine the following using calculus: a curve’s arc length, volume and surface area of a solid of revolution [9-11].

An example of a work of architecture (Figure 1a) whose volume and surface area was calculated using these formulae is also presented. The first and fundamental step is to determine the shape of the rotation path of the solid of revolution (Figure 1b) and to adopt a set of formulae, whose plot fits this curve (Figure 1c).

$$f(x)=\begin{cases} -1.381x - 0.048 & ; \quad x\in[-8;-5] \\ \frac{-2}{7}(x+\frac{7}{2})^2+\frac{15}{2} & ; \quad x\in[-5;0] \\ \frac{4}{25}(x-5)^2 & ; \quad x\in[0;5] \end{cases}$$

a) b) c)

Figure 1: Determine surface area and volume of a solid of revolution with the shape and proportions close to those of the dome above the garden gate at the church of the Birth of the Blessed Virgin Mary in Harklowa, Poland: a) photograph of the object; b) curve fitting the rotation path of the solid of revolution; c) the formulae describing the adopted curve (Source: authors).

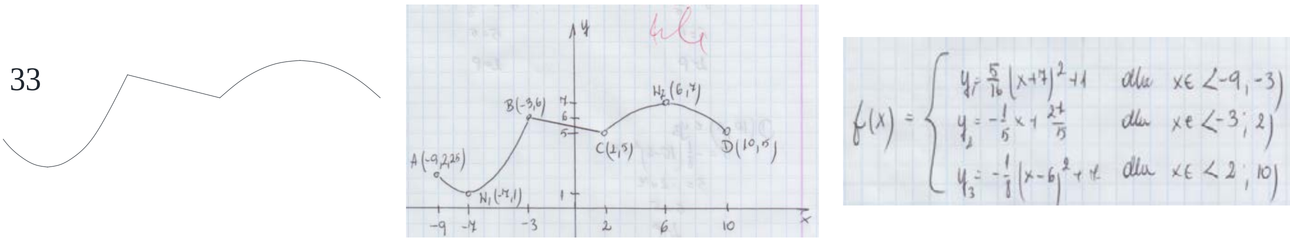
The first stage is associated with handing out and discussing the project, and often takes place in the middle of the semester. It is associated with the mathematical problems tackled during classes, which are associated with studying a function’s continuity. They are defined with four elementary functions determined by sections. The task of the students is not only to analyse the continuity of a function, but also to plot it. The function f(x) (unrelated to the f(x) in Figure 1) is an example that illustrates this type of procedure:

$$
f(x)=\begin{cases} arctg(x+\frac{\pi}{2}); & x<-\frac{\pi}{2} \\ \cos x; & -\frac{\pi}{2}\le x\le\frac{\pi}{2} \\ -tgx; & \frac{\pi}{2}<x<\pi \\ -1; & \pi<x \end{cases} \tag{1}
$$

In reference to these types of mathematical problem and the lecture, each student receives an individual project subject, which presents a curve (example in Figure 2a). The task the students perform is atypical, as it is a reversal of their previous experiences. The results of studying reverse thinking among mathematics students can be observed in the works of Maf'ulah et al who presented the main operations on reciprocals in algebra:

> *Effective algebraic thinking sometimes involves reversibility (i.e. being able to undo mathematical processes, as well as to do them). In effect, it is the capacity not only to use a process to reach a goal, but also to be able to understand the process well enough to work backwards from the answer to the starting point* [12].

This time, the plot is given and the formula of the function needs to be determined. The element that makes it easier is the curve's location within the co-ordinate system (above the x axis) (Figure 2b). In a subsequent presentation, students are shown how to describe curves using three quadratic functions and a single linear function. Assuming the co-ordinates of the vertices of parabolas and common points to be integers makes it easier to quickly plot the canonical form of a quadratic function. Passing on to the next stage of the project is possible only after the instructors verify the formulae proposed by students.



a) b) c)

Figure 2: Individual theme of the poster: a) the given curve; b) curve located in the co-ordinate system; c) formulae describing the curve. (Sources: Krystyna Romaniak and student Kamil Federyga).

In the next step, students perform calculations using calculus. Their task is to calculate at least two values out of four. These can be: surface area between the x axis and the curve plot; the length of the curve arc; volume or surface area of the solid of revolution created as a result of rotating the curve around the x axis. Students can use mathematical software when performing their calculations. During classes they receive information on how to use Mathematica software from the WolframAlpha Web site [13].

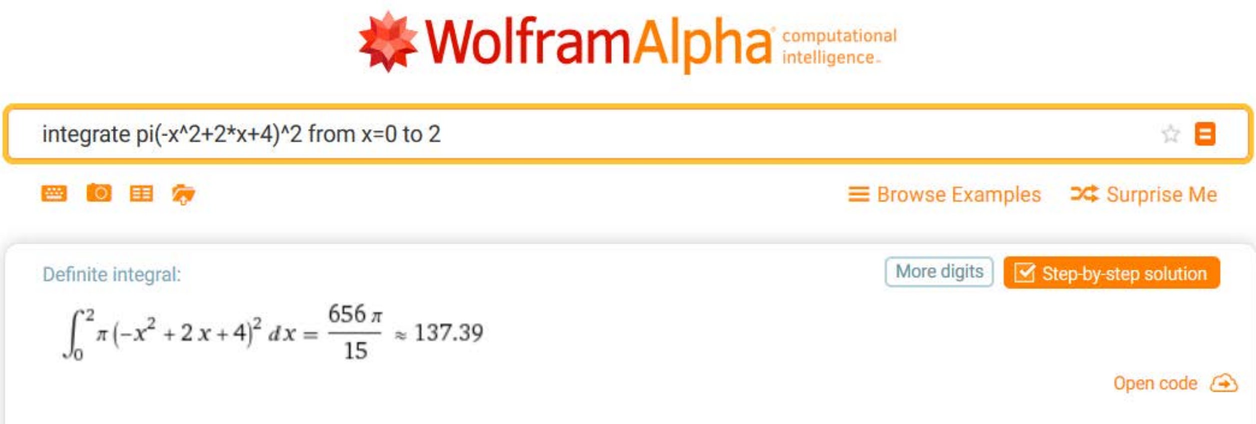
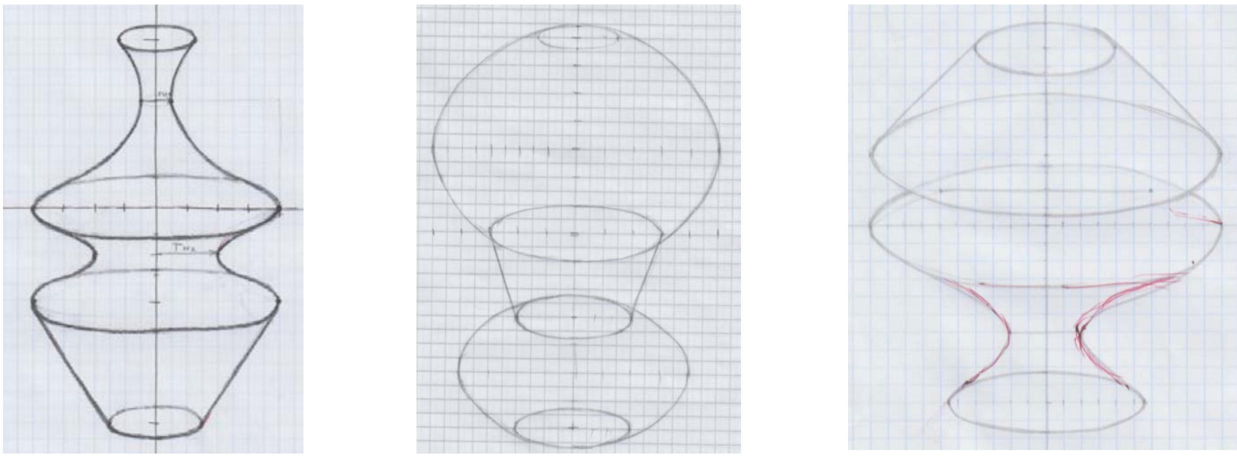


Figure 3: Calculating the volume of a solid of revolution using Mathematica software, available on the WolframAlpha Web site.

The subsequent stage to complete the poster is a hand-drawn sketch of the solid of revolution created as a result of the revolution of the curve around the x axis. Instructors present the manner of drawing the sketch in axonometric projection using the dimensions adopted in the co-ordinate system (Figure 4). They plot an ellipse using the cuboid method. Students can use it, but it is not mandatory. The sketch of the solid of revolution is corrected by instructors during classes.



a) b) c)

Figure 4: Hand-drawn sketches of the solid created by rotating the curve around the x axis (Sources: student works - Aneta Kozińska, Helena Hoffmann and Wojciech Dziaduła).

The final stage is manually preparing a poster. A stiff A4 format sheet of Bristol paper (drawing paper glued together to create multi-ply sheets) is required. The poster's main elements are a plot of the curve in the co-ordinate system, formulae of the functions that describe the curve, calculations employing calculus (surface area between the x axis and the curve plot, curve arc length, volume or surface area of the solid of revolution), and a drawing of the solid created as a result of rotating the curve around the x axis. The colour scheme and technique to draw the poster are left to the students' discretion (black and white, colour, in pencil, crayons, pastels, watercolours or a collage of various materials etc). The analytical calculations on the poster must be correct, which is why corrections are necessary if there are any mistakes. The final grade is associated with the project's aesthetic quality, the correctness of drawing the solid of revolution and the concept behind the poster.

Figure 5: Poster for the Mathematics module prepared by students of the first year of the architecture course, first semester, 2019/2020 academic year (Source: student works - Małgorzata Kołodziej, Ewelina Kordek and Błażej Kmonk).

CONCLUSIONS

The project presented in the article is prepared in multiple stages over the course of several meetings. These are most often presentations which supplement other types of class, such as tests. Only the freehand sketch of the solid is performed at a separate meeting - which is one of the final meetings in the first semester. All the information concerning the poster is posted on the e-learning platform, therefore students can return to it at any time. This makes it easier to complete the project's successive elements, which are ultimately placed on the poster, as a part of a homework assignment.

Despite being given a lot of advice, students must demonstrate creativity and prepare the poster according to their own vision. They can adopt any formula that has a plot concordant with the drawing of the curve during the stage of determining the function. These are not necessarily always quadratic functions. The greatest variation is demonstrated during the end stage, as these are usually unique works that surprise with their colour and technique. The subject, presented in graphical form, is a concretisation of a number of fundamental objectives:

- it leads to visually solving problems and the presentation of the work of an architect;
- it presents a practical aspect of calculus and demonstrates the possibilities of its use in the professional work of an architect;
- it engages students in the process of solving architectural problems using mathematical tools;
- it leads to calculations that utilise knowledge gained during classes and the use of mathematical software;
- it integrates mathematics with other modules: freehand drawing (the project is performed manually) and descriptive geometry (the use of axonometric or perspective projection);
- it refers to knowledge gained throughout the entire semester.

The scheme of teaching mathematics at the Faculty of Architecture is constantly changing due to progress associated with the tools and software used in education. Fundamental content forms the core of teaching, one that is not subjected to dynamic transformation. However, the manner of transferring knowledge is always adapted to the specialisation of students, as well as the technological capabilities of both universities and students.

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